TRAVAUX PRATIQUES DE

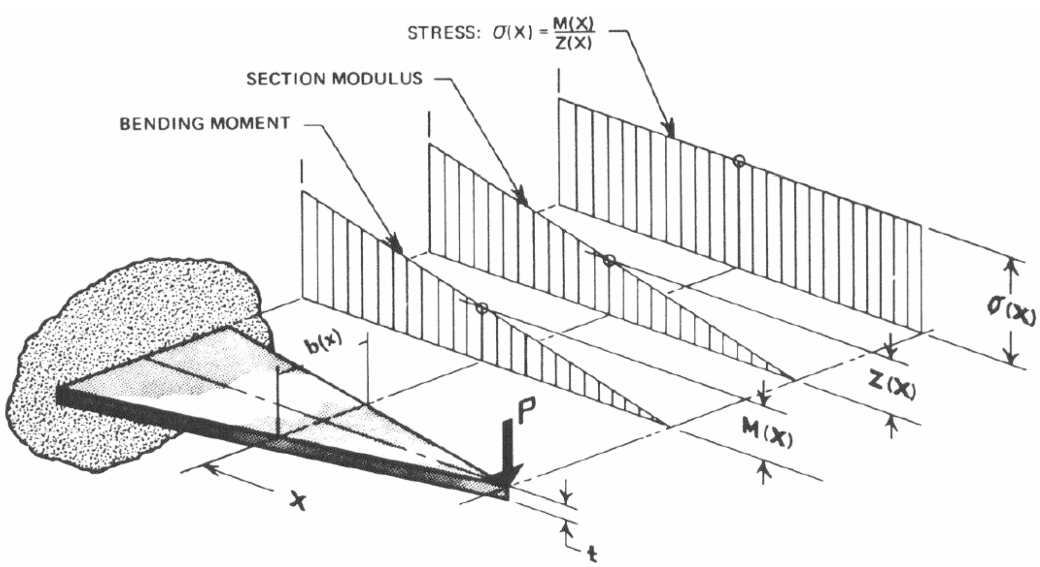
DIMENSIONNEMENT DES STRUCTURES

Techniques d'extensométrie

TP n° 4 :

Poutre à contrainte constante

Constant-Stress Beams



E-106A CONSTANT STRESS BEAMS

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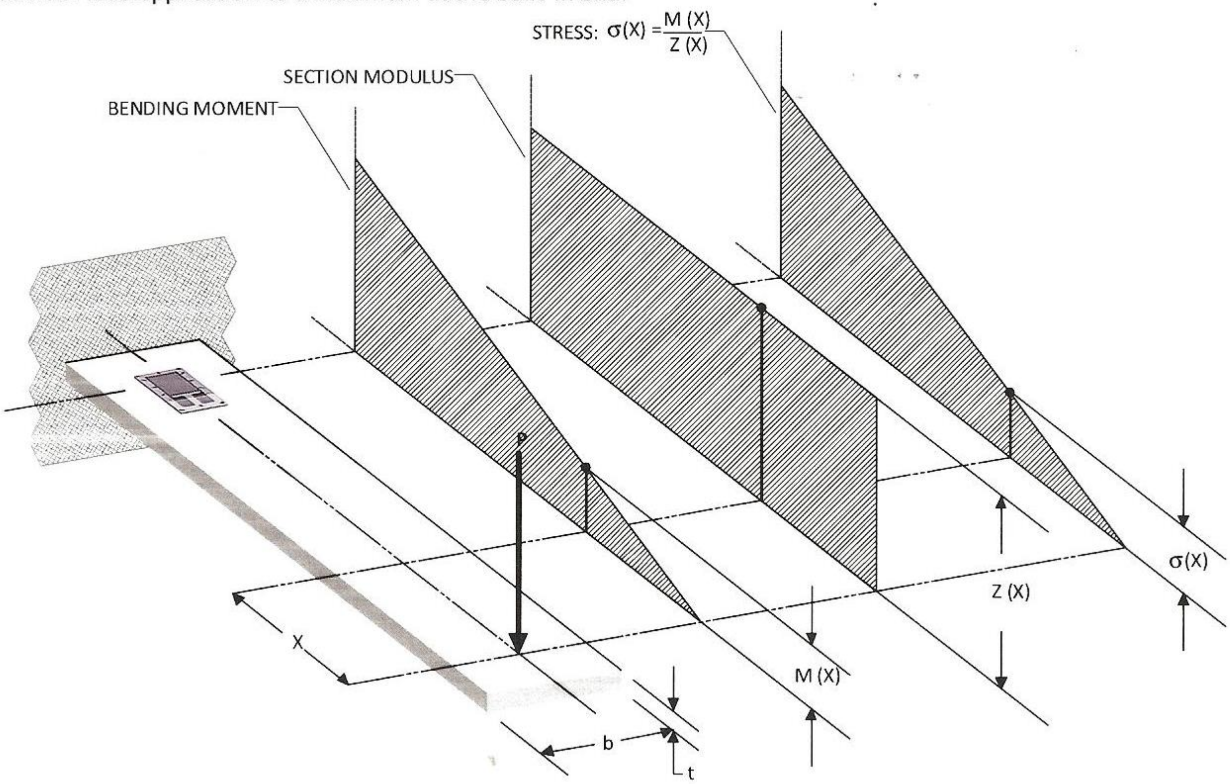
I. INTRODUCTION

In a cantilever beam with a single point load at the free end, the bending moment varies linearly from zero at the point of load application to a maximum at the built-in end.



For a parallel-sided beam, as illustrated in the preceding sketch the axial stress on the beam surface is proportional to the bending moment, and is expressed by:

$$\sigma(X) = \frac{M(X) \bullet c}{I} = \frac{6PX}{bt^2} = \frac{PX}{Z} \quad (1)$$

where:

- σ(X) = bending stress on beam surface at distance X from point of load application, psi (N/m^2)
- M(X) = bending moment at distance X, in-lbs (mN)
- P = load, lbf (N)
- c = $\frac{t}{2}$ = distance from neutral axis to beam surface, in (m)
- I = $\frac{bt^3}{12}$ = moment of inertia of beam cross section, in^4 (m^4)
- Z = $\frac{bt^2}{6}$ = section modulus of beam, in^3 (m^3)
- b = Beam width, in (m)
- t = Beam thickness, in (m)

From Eq. (1), it is apparent that the stress varies linearly from zero at the point of load application to a maximum at the built-in end, if the beam is parallel-sided so that the section modulus, Z, is constant. The surface of the cantilever beam is in a uniaxial stress state, and from Hooke's Law for this case,

$$\varepsilon(X) = \frac{\sigma(X)}{E} = \frac{PX}{EZ} \quad (2)$$

The parallel-sided beam does not generally represent the optimum utilization of material. When the beam is designed to provide the desired margin of safety between the maximum applied stress and the allowable stress for the beam material, all but the built-in end of the beam is under stressed. The ideal beam configuration, in terms of stress distribution, would be such that the stress is uniform from one end to the other.

From Eq. (1), it can be seen that, for any predetermined load, P, if σ(X) is to be constant while X varies from zero to L, the section modulus must vary directly with X, so that:

$$\frac{X}{Z(X)} = \frac{6X}{b(X)[t(X)]^2} = \text{constant} \tag{3}$$

That is, the width and/or the thickness of the beam must be functions of X and vary along the length of the beam. For example, if the width is constant, and the thickness varies as the square root of X,

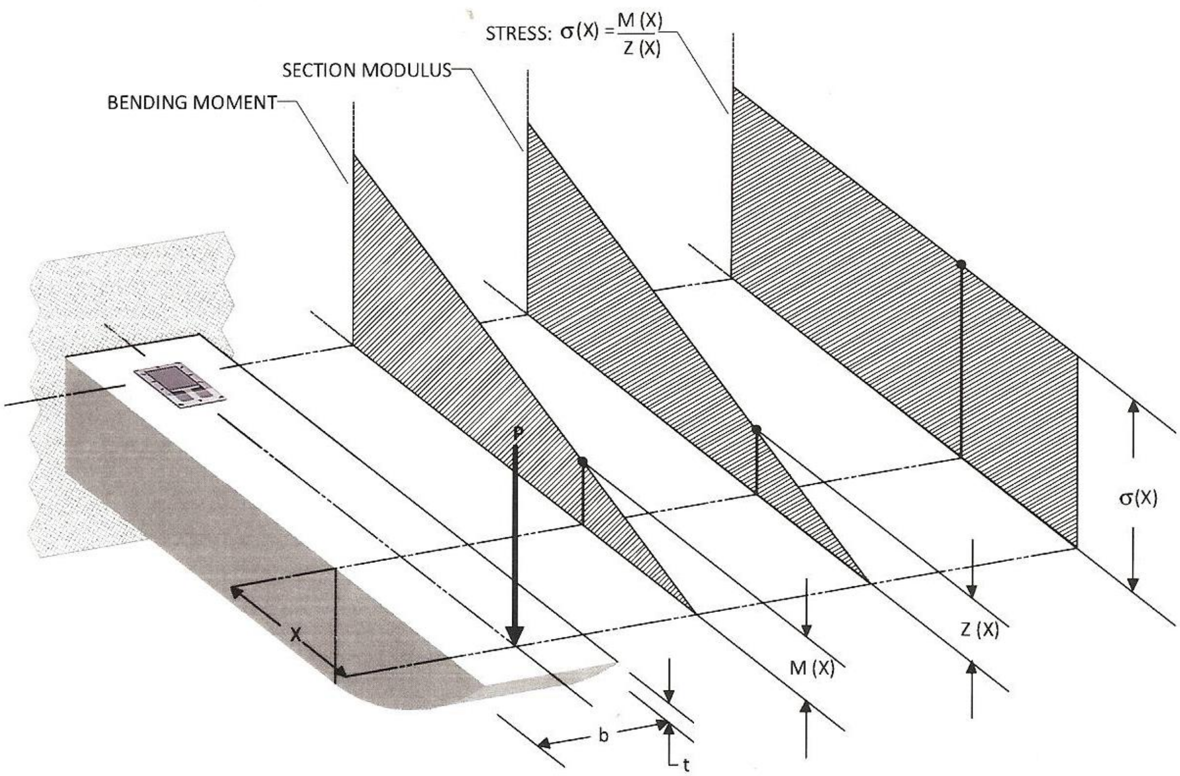
$$t(X) = K_1\sqrt{X} \ \ (\mathrm{b} = \text{constant})$$

$$Z(X) = \frac{bK_1{}^2X}{6} \tag{4}$$

and,

$$\sigma(X) = \frac{6PX}{bK_1{}^2X} = \frac{6P}{bK_1{}^2} = \text{constant} \tag{5}$$

Thus, for this configuration the stress is constant from one end of the beam to the other (except for localized effects in the vicinity of the loading point and the built-in end; which are not considered here). The constant-stress cantilever beam we have just designed is illustrated below:



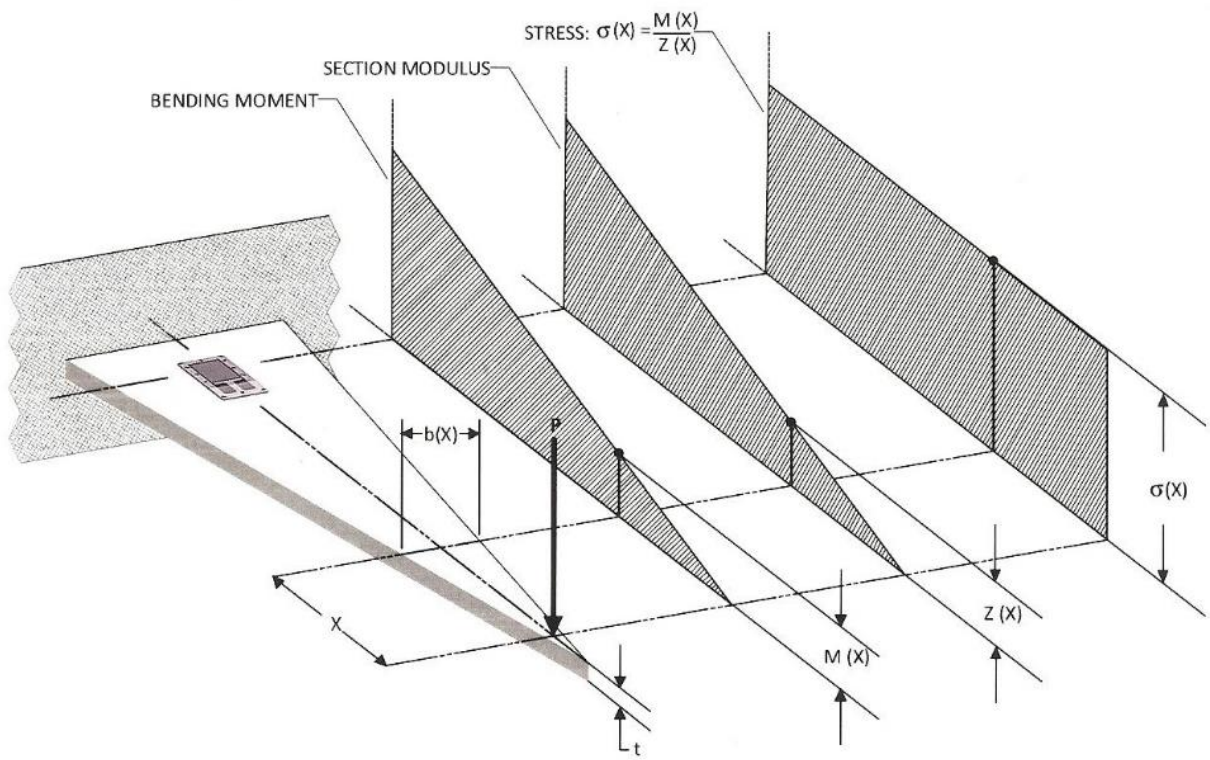
Because a beam which varies in thickness according to Eq. (5) is generally both difficult and expensive to manufacture, it is more common to vary the width to achieve constant bending stress along the beam length. The section modulus also can be made proportional to X by making the width proportional to X, and holding the thickness constant. That is,

$$b(X) = K_2 X \ \ (t = \text{constant}) \tag{6}$$

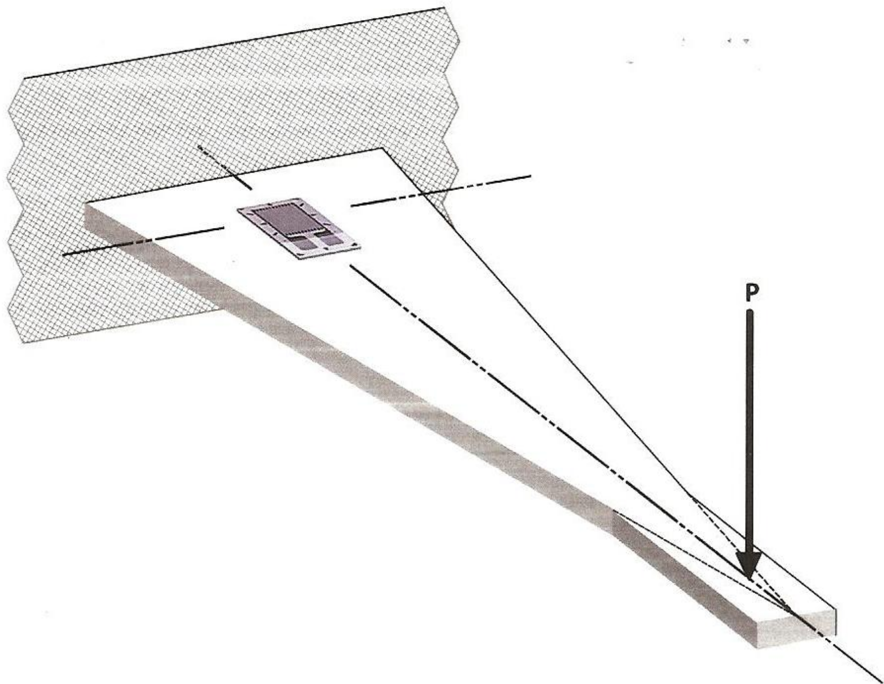
and,

$$\sigma(X) = \frac{6PX}{K_2 X t^2} = \frac{6P}{K_2 t^2} = \text{constant} \quad -\text{similarly}- \quad \varepsilon(X) = \frac{6P}{E K_2 t^2} = \text{constant} \tag{7}$$

The beam now has the following appearance:



The beam shown on the previous page is not practicable because there is negligible material directly under the load to support the vertical shear force. This condition can be remedied, while retaining the constant-stress character of the beam over most of its length, by the following design:



Note that the sides of the beam in the constant-stress section must still converge toward the point of load application to satisfy Eq. (7) over this region.

The primary purpose of this experiment is to verify the relationship among the bending-moment, section-modulus, and stress (or strain) distributions along the length of a cantilever beam. A second purpose of the experiment is to demonstrate the practical application of this relationship to the design of "efficient" beams. From the viewpoint of material utilization, the beam design is ordinarily optimal when the greatest practicable volume of material in the beam is subjected uniformly to the maximum design stress. For a beam with a rectangular cross-section, this leads to the concept of the constant-stress beam in which the surface stress is everywhere uniform. While the constant-stress beams described in this experiment are all cantilever beams, it should be noted that the same general principles apply to the design of any beam under any mode of plane loading, since the bending-moment/section-modulus/stress relationship applies to all beams.

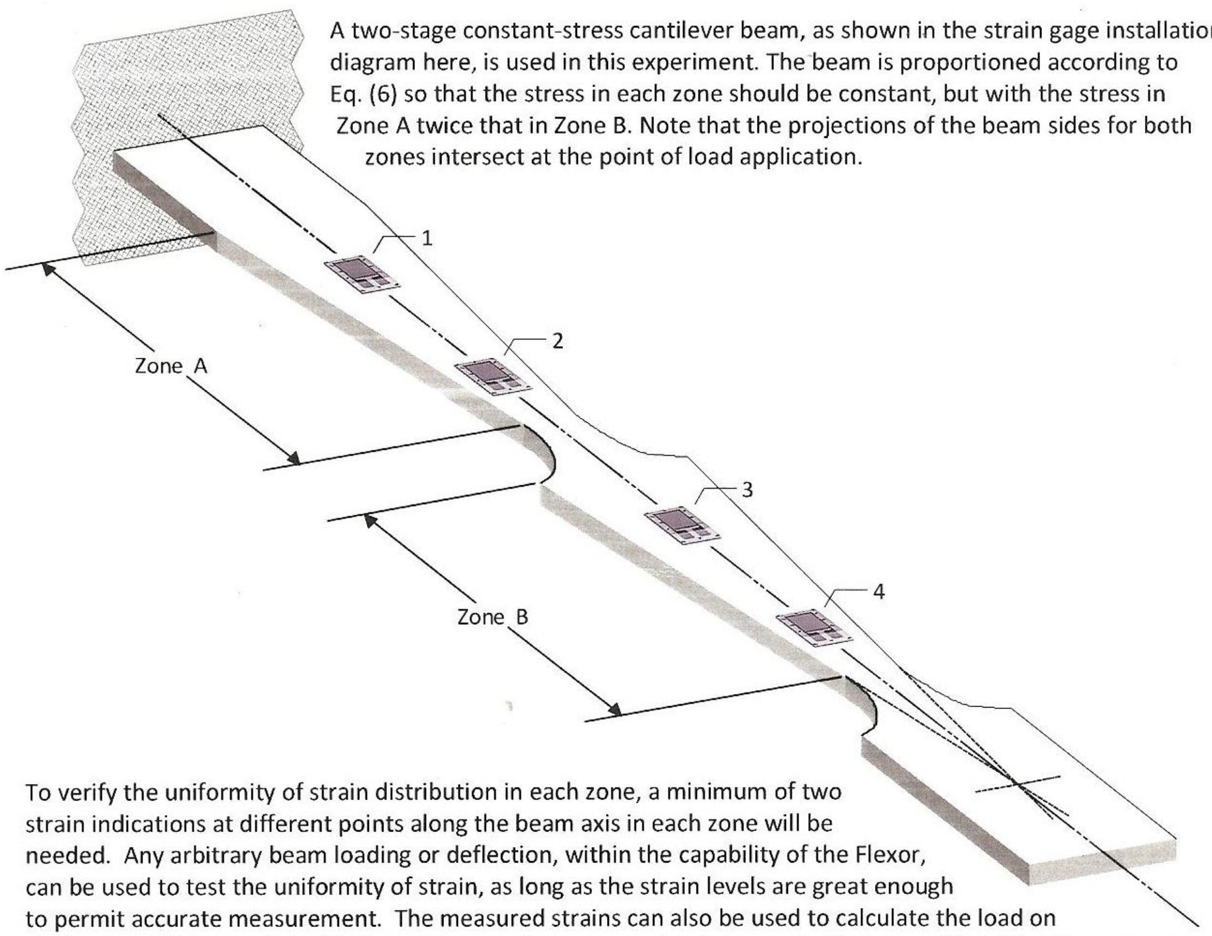
II. EQUIPMENT AND SUPPLIES

- Flexor, cantilever flexure frame
- High-strength aluminum alloy pre-gaged beam, B-106A, 1/4 x 1 x 12-1/2 *(6 x 25 x 320 mm)*
- Model P3 Strain Indicator, D4 Data Acquisition Conditioner or equivalent
- Micrometer or Vernier Caliper
- Drafting or Machinist's Scale

III. PROCEDURE

A. GENERAL THEORY

A two-stage constant-stress cantilever beam, as shown in the strain gage installation diagram here, is used in this experiment. The beam is proportioned according to Eq. (6) so that the stress in each zone should be constant, but with the stress in Zone A twice that in Zone B. Note that the projections of the beam sides for both zones intersect at the point of load application.



To verify the uniformity of strain distribution in each zone, a minimum of two strain indications at different points along the beam axis in each zone will be needed. Any arbitrary beam loading or deflection, within the capability of the Flexor, can be used to test the uniformity of strain, as long as the strain levels are great enough to permit accurate measurement. The measured strains can also be used to calculate the load on the beam, utilizing Eq. (7) with the section modulus appropriately expressed in terms of Eq. (6). Measuring the beam width at any distance X from the point of load application, K_2 can be determined from Eq. (6). Substituting K_2, the measured thickness, and the observed strain into Eq. (7), along with the elastic modulus of the beam material, permits solving for the load, P.

B. STRAIN GAGE CONCEPTS

Micro-Measurements foil strain gages are intrinsically temperature-compensated for use on a material with a particular thermal coefficient of expansion. Because of this, the four strain gages employed in this experiment can be used individually in a “quarter-bridge arrangement”, completing the bridge circuit each time with the 120-ohm precision resistor built into the Strain Indicator. For quarter-bridge operation a “three-wire” circuit is ordinarily used for each gage in order to obtain leadwire compensation by placing equal lengths of leadwire in adjacent arms of the bridge circuit.

A strain gage indicates the average strain under the grid area when placed in a non-uniform strain field. In this case, the strain field is expected to be uniform, or nearly so, and the strain indicated by each gage is equal to the strain at the gage centerline.

C. ACQUISITION OF DATA

Flexure Setup:

- Back the micrometer / loading screw out; in order to allow the beam access to be placed in the Flexor
- Insert the beam into the Flexor with the surface containing the strain gages facing up
    - The micrometer should not be touching the beam at this point
- Slide the free end of the beam completely under the clamping plate that is attached to the tightening knob
- Firmly clamp the beam into place with the tightening knob
- Connect the leadwires from the Strain Gages to the Strain Indicator according to the Wiring Diagram on page 8
    - Connect Strain Gage 1 to Channel 1 with the red lead to P+, white lead to S- and Black lead to D120.
    - Connect Strain Gages 2,3 and 4 to the appropriate channels

Note: All four of the strain gages will be connected to the strain indicator at the same time

Strain Indicator Setup

- Power on the strain indicator
- Set the gage factor on all four channels to the value given on B-106A beam
    - *Do not adjust the gage factor setting again during the experiment*
- Set each channel to operate as a quarter-bridge
- With the Flexor micrometer still clear of the beam, balance the strain indicator until the reading indicates precisely 0 for each of the four gages
- The beam zero-deflection reading of all the gages should be recorded on the "Work Sheet" as 0με.
    - *Do not adjust the balance control again during the experiment.*

Note: The gages will be connected to the strain indicator with the beam undeflected, and again with the beam deflected, to obtain two sets of readings for determining the strains.

Testing

- Deflect the beam by rotating the micrometer spindle clockwise, until Channel 1 on the strain indicator readout registers 1000με
    - Channel 1 of the strain indicator should correspond with Strain Gage 1
- Record the values for Strain Gages 1, 2, 3 and 4 on the "Work Sheet"

Verification

As a check on the stability of the system, back the Flexor loading screw away until it clears the beam. The strain indicator readout should now read very close to zero for all four channels if the system is operating normally. If the number is more than 10με or so from zero, the source of the error should be located, and the experiment performed again.

Pre-gaged beams supplied by Micro-Measurements have been tested for gage stability at the time of manufacture, and should perform in a highly repeatable manner unless one or more of the gages has been damaged. If the zero-beam-deflection readings of the strain indicator fail to repeat well, the connections may not have been snug enough to avoid small contact resistance changes when the wires are moved. The connections should be secure enough to allow a "wiggle test" of the leadwires without a significant zero balance shift.

IV. ANALYSIS AND PRESENTATION OF DATA

Strains in Zone (A)

Refer to the "Work Sheet". Review the final indications for Gages 1 and 2, at the maximum deflection in the table on the "Work Sheet". Within the limits of experimental error, the resulting strains should be nearly the same if this zone of the beam is, in fact, characterized by constant stress. The average of these two strains is the best estimate of the strain in Zone A. Afterwards, record the resulting average value in the Average Strains section of the "Worksheet".

Strains in Zone (B)

Review the final indications for Gages 3 and 4 at the maximum deflection and record the results in the table on the "Work Sheet". Again, the two strains should be very nearly equal because of the constant-stress beam shape in this region. Average these values as the best estimate of the strain in Zone B. Again, record the result in the Average Strains section of the "Worksheet".

Comparison of Strain and Section Moduli

Compare the averages obtained in the two preceding steps by calculating the ratio of the average strain in Zone A to the average strain in Zone B (i.e. $\varepsilon_A / \varepsilon_B$). Measure the beam width at the centerlines of Gages 1 and 3, and the corresponding distances from the point of load application to Gages 1 and 3. Substitute these pairs of measurements into the "Comparison of Strain and Section Moduli" section of the "Work Sheet" to calculate K_{2A} and K_{2B}. Compare K_{2B} / K_{2A} to $\varepsilon_A / \varepsilon_B$.

Calculation of Load

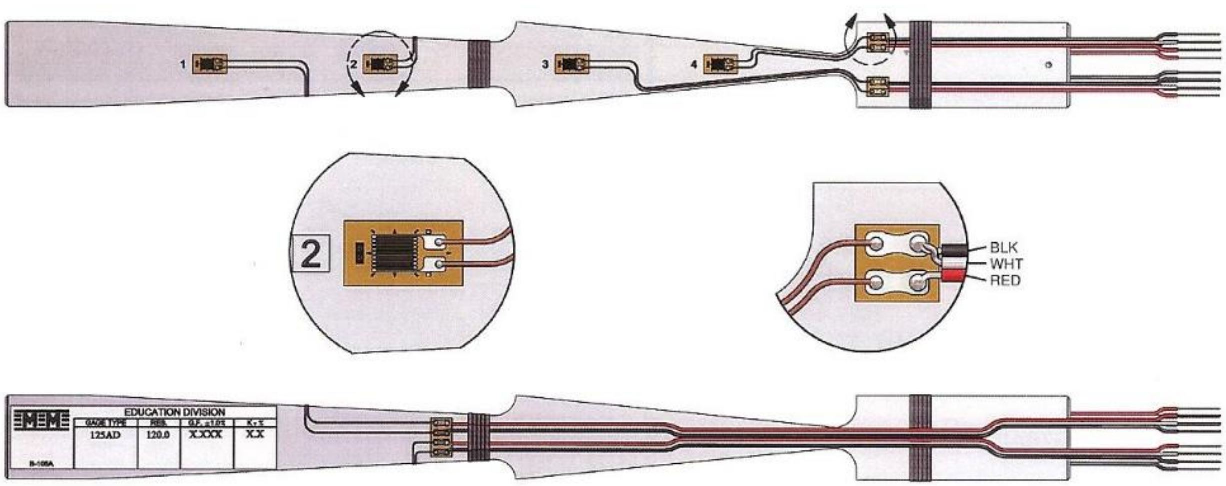
Within the "Computation of Load" section of the "Work Sheet", assuming the modulus of elasticity of the beam material is 10.4×10^6 psi *(7.17×10^{10} N/m^2)*, calculate the beam load, first with K_{2A} and ε_A, then with K_{2B} and ε_B, and compare the results.

V. REPORT

- Prepare a brief report in your own words:
    - Describe the purpose of the experiment, the equipment / setup used, and the results obtained
        - Include all original data and calculations
    - Explain the probable sources of error in the experiment, and their relative effects on the accuracy of the stress concentration factor you have obtained
        - Analyze, for example, the error due to locating the point of load application at locations other than the intersection of the beam side projections
    - Design a constant-stress cantilever beam in which both the thickness and width vary continuously along the length. In other words, select pairs of simple compatible functions for b(X) and t(X) so that Z(X) = KX

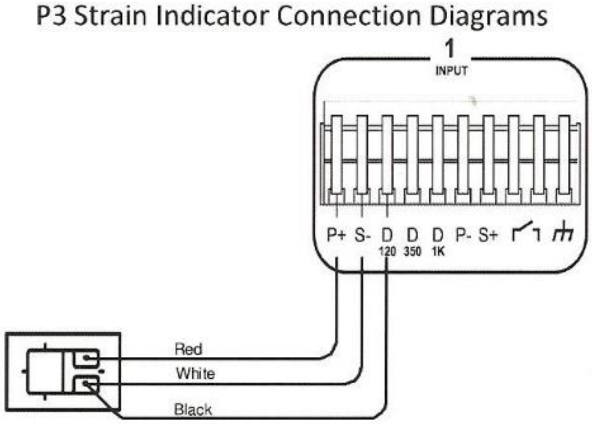
APPENDIX

I. FLEXOR BEAM WITH PRE-ATTACHED STRAIN GAGES (1, 2, 3 & 4)

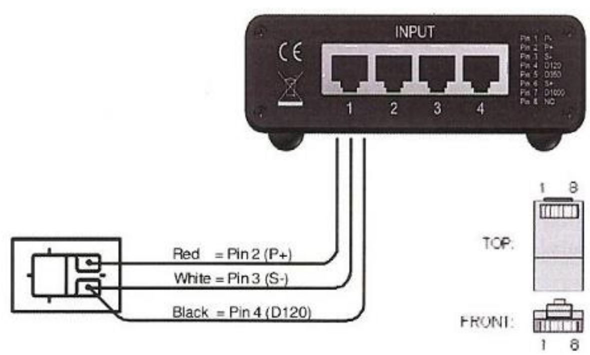


II. WIRING DIAGRAMS

P3 Strain Indicator Connection Diagrams



D4 Strain Indicator Connection Diagrams



III. WORK SHEET

STRAIN MEASUREMENTS: Gage Factor of Strain Gages 1, 2, 3 and 4 ________________

GAGE	INITIAL READING	MEASURED STRAIN	
1	0 με	1000 με	Zone A
2			
3			Zone B
4			

AVERAGE STRAINS:

$$\varepsilon_A = \frac{\varepsilon_1 + \varepsilon_2}{2} = \frac{(\qquad) + (\qquad)}{2} = \_\_\_\_\_\_\_\_\_\_ \mu\varepsilon$$

$$\varepsilon_B = \frac{\varepsilon_3 + \varepsilon_4}{2} = \frac{(\qquad) + (\qquad)}{2} = \_\_\_\_\_\_\_\_\_\_ \mu\varepsilon$$

COMPARISON OF STRAIN LEVELS AND SECTION MODULI:

STRAIN RATIO:

$$\frac{\varepsilon_A}{\varepsilon_B} = \frac{(\qquad)}{(\qquad)} = \_\_\_\_\_\_\_\_\_\_$$

BEAM DIMENSIONS:

Widths: b_1 - ________________ in. (m); b_3 - ________________ in. (m)

Distances: X_1 - ________________ in. (m); X_3 - ________________ in. (m)

SECTION MODULUS RATIO:

$$\frac{Z_B}{Z_A} = \frac{\frac{K_{2B}t^2}{6}}{\frac{K_{2A}t^2}{6}} = \frac{K_{2B}}{K_{2A}} = \frac{b_3 X_1}{b_1 X_3} = \frac{(\qquad)(\qquad)}{(\qquad)(\qquad)} = \_\_\_\_\_\_\_\_\_\_$$

SECTION CONSTANTS:

$$K_{2A} = \frac{b_1}{X_1} = \frac{(\qquad)}{(\qquad)} = \_\_\_\_\_\_\_\_\_\_ \qquad K_{2B} = \frac{b_3}{X_3} = \frac{(\qquad)}{(\qquad)} = \_\_\_\_\_\_\_\_\_\_$$

COMPUTATION OF LOAD:

$$P = \frac{\varepsilon_A K_{2A} E t^2}{6} = \frac{(\qquad)x10^{-6}(\qquad)(\qquad)(\qquad)^2}{6} = \_\_\_\_\_\_\_\_\_\_ \text{lbf (N)}$$

$$P = \frac{\varepsilon_B K_{2B} E t^2}{6} = \frac{(\qquad)x10^{-6}(\qquad)(\qquad)(\qquad)^2}{6} = \_\_\_\_\_\_\_\_\_\_ \text{lbf (N)}$$